Lab: AP Review Sheets — AP PHYSICS

Chapter 4: Motion in Two Dimensions — by Justin Wang

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**Background / Summary**

Motion in two dimensions refers to the movement of an object in both the horizontal and vertical directions simultaneously, typically analyzed using concepts like vectors, projectile motion, and relative motion, fundamental to understanding complex trajectories and interactions in physics.

**Vector Representation**

- Represent vectors using notation like $\vec{V}$ for velocity, $\vec{a}$ for acceleration, and $\vec{d}$ for displacement.
- Use vector addition and subtraction to find resultant vectors
- Equations:
  - $$\vec{V} = \frac{\Delta\vec{d}}{\Delta t}$$
    $$\vec{a} = \frac{\Delta\vec{V}}{\Delta t}$$

**Projectile Motion**

- Break motion into horizontal and vertical components.
- Use kinematic equations separately for horizontal and vertical motion.
- Vertical motion is uniformly accelerated due to gravity
- Equations:
  - $$x = v_{0x}t$$
    $$y = v_{0y}t - \frac{1}{2}gt^2$$
    $$v_y = v_{0y} - gt$$
    $$v_{fy}^2 = v_{iy}^2 + 2g\Delta y$$

# To solve 2-d problems:

- ***Strategy 1***
  Break given vectors into **x-** and **y-**components; then apply kinematics in **x** and **y** directions separately. Finally, resolve components as needed for final solution (in polar notation?)
- ***Strategy 2***
  Using vector-based kinematic equations (like $\Delta\mathbf{r}=\mathbf{v}_i t+(1/2)\mathbf{a}t^2$) with vectors written in unit-vector (**i, j**) notation.

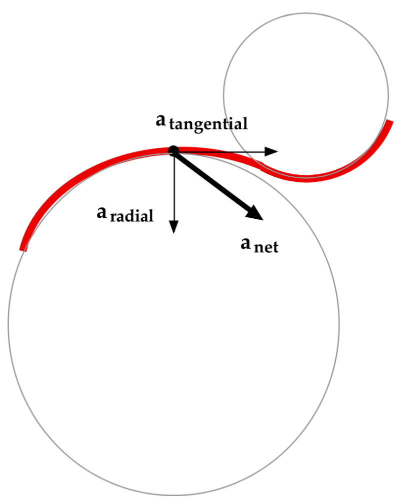
**Circular Motion:**

- Understand that centripetal acceleration acts towards the center of the circular path.
- Use centripetal force to keep objects in circular motion.
- Equations:
  - $$a_c = \frac{v^2}{r}$$

*Radial acceleration* $\mathbf{a}_r$ is due to the change in direction of the velocity vector: $\mathbf{a}_r=v^2/r$

*Tangential acceleration* $\mathbf{a}_t$ is due to the change in speed of the particle: $\mathbf{a}_t = dv/dt$.

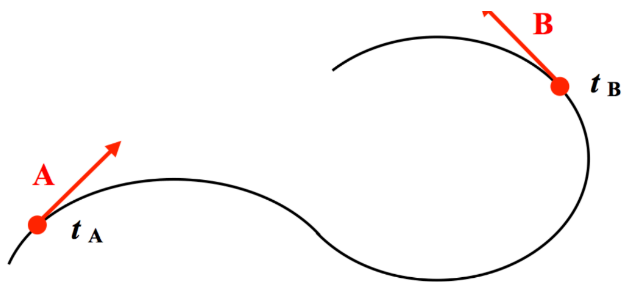
$$\vec{\mathbf{a}} = \vec{\mathbf{a}}_t + \vec{\mathbf{a}}_r$$



**Questions:**

1) A projectile is fired horizontally from a height of 20 meters above the ground, with an initial velocity of 7.0 m/s. How far does the projectile travel horizontally before it reaches the ground?

   a. 7m
   b. 14m
   c. 140m
   d. 3.5m
   e. 20m

2) 


An object travels along a path shown above, with changing velocity as indicated by vectors **A** and **B**. Which vector best represents the net acceleration of the object from time $t_A$ to $t_B$?

a. 

b. 

c. 

d. 

e. 

3) A circus cannon fires an acrobat into the air at an angle of 45° above the horizontal, and the acrobat reaches a maximum height $y$ above her original launch height. The cannon is now aimed so that it fires straight up into the air at an angle of 90° to the horizontal. What is the maximum height reached by the same acrobat now?

**Solutions:**

1)

**Answer:**

The correct answer is *b*. We begin by finding how much time it takes the object to fall the 20m:

$$\Delta y = v_i t + \tfrac{1}{2}at^2$$

$$-20m = 0t + \tfrac{1}{2}(-10)t^2$$

$$t = \sqrt{4} = 2\ \ s$$

Then, determine how far the ball travels horizontally during that time:

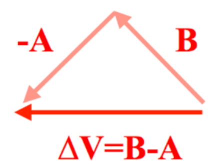
$$\Delta x = v_x t$$

$$\Delta x = (7m/s)(2s) = 14m$$

2)

**Answer:**

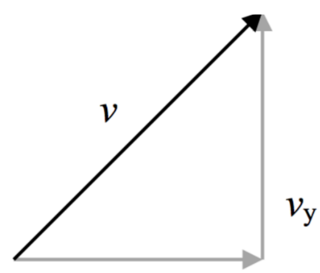
The correct answer is *d*. The direction of acceleration is the same as the direction of the change in velocity, according to $a = \frac{v_f - v_i}{t}$. Because $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$, we can determine $\Delta v$ graphically by adding $\mathbf{v}_f$ to the negative of $\mathbf{v}_i$ , or B+(-A). Placing the B vector "tip-to-tail" with the –A vector gives a direction for $\Delta \mathbf{v}$ (and therefore, **a**) to the left.




3) The acrobat reaches her height in the first instance based on the initial vertical component of velocity, $v_y$:

$$v_f^{\ 2} = v_i^{\ 2} - 2ay$$

$$y = \frac{0 - v_i^{\ 2}}{-2g} = \frac{v_i^{\ 2}}{2g}$$




For the second situation, the vertical velocity $v$ is greater than $v_y$ from before, by a factor of $\sqrt{2}$. Using this information:

$$y' = \frac{(v_i')^2}{2g}$$

$$y' = \frac{(v\sqrt{2})^2}{2g} = \frac{2v^2}{2g} = 2y$$